Section 5.1

1) $\int 3x^2 dx$

$$= 3\int x^2 dx$$

1st: $\int af(x)dx = a\int f(x)dx$

$$= \cancel{3} \cdot \frac{1}{\cancel{3}} x^3 + C$$

$$= x^3 + C$$

2nd: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ $\quad provided\ \overset{n}{\cancel{x}} \neq -1$

$Answer\ \ x^3 + C$

3) $\int \frac{1}{3} x dx$

1^{st}: $\int af(x)dx = a \int f(x)dx$

$$= \frac{1}{3} \int x^1 dx$$

$$= \frac{1}{3} \cdot \frac{1}{2} x^2 + C$$

2^{nd}: Power Rule: $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ \cancel{x}\, n \neq -1$

$$= \frac{1}{6} x^2 + C$$

Answer: $\frac{1}{6} x^2 + C$

5) $\int 2dx$ $= 2x + C$

6) $\int 7dx$

1st: Integral of a constant Rule: $\int adx \;=\; ax + C \quad (a\ is\ any\ real\ number)$

Answer: $2x + C$

7) $\int (6x+5)dx$

$$= \int 6x\,dx + \int 5\,dx$$

$$= 6\int x\,dx + \int 5\,dx$$

$$= \cancelto{3}{6}\cdot\frac{1}{\cancelto{1}{2}}x^2 + 5x + C$$

$$= 3x^2 + 5x + C$$

1st:

$\int \big(f(x)+g(x)\big)dx = \int f(x)dx \;\; + \int g(x)dx$

$\int \big(f(x)-g(x)\big)dx = \int f(x)dx \;\; - \int g(x)dx$

2nd: $\int af(x)dx = a\int f(x)dx$

3rd:

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \;\; provided\; n \neq -1$

Integral of a constant Rule: $\int a dx \; = \; ax + C \;\; (a\ is\ any\ real\ number)$

Answer: $3x^2 + 5x + C$

9) $\int \frac{5}{x^2} dx$

1st: Rewrite with negative exponent $= \int 5x^{-2} dx$

2nd: $\int af(x)dx = a\int f(x)dx$ $= 5\int x^{-2} dx$

$(-2+1=-1$ NEW Exponent$)$

$= 5 \cdot \frac{1}{-1} x^{-1} + C$

3rd: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ *provided* $n \neq -1$

$= 5 \cdot -1\, x^{-1} + C$

4th: rewrite with positive exponent

$= -5x^{-1} + C$

$= -\frac{5}{x} + C$

Answer: $-\frac{5}{x} + C$

11) $\int \frac{3}{x^4} dx$

1st: Rewrite with negative exponent $= \int 3x^{-4} dx$

2nd: $\int af(x)dx = a\int f(x)dx$ $= 3\int x^{-4} dx$

$= 3 \cdot \frac{1}{-3} x^{-3} + C$

3rd: Power Rule: $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ n \neq -1$

$= -1\, x^{-3} + C$

$\frac{3}{1} \cdot \frac{1}{-3} = \frac{3}{-3} = -1$

$= -\frac{1}{x^3} + C$

4th: rewrite with positive exponent

Answer: $-\frac{1}{x^3} + C$

13) $\int 2x(x^2+3)dx$

$2x \cdot x^2 \quad 2x \cdot 3$

1st: Rewrite by clearing parenthesis $= \int (2x^3+6x)dx$

2nd:

$\int (f(x)+g(x))dx = \int f(x)dx \quad + \int g(x)dx$ $= \int 2x^3 dx + \int 6x dx$

$\int (f(x)-g(x))dx = \int f(x)dx \quad - \int g(x)dx$

$= 2\int x^3 dx + 6\int x dx$

3rd: $\int af(x)dx = a\int f(x)dx$

$= \cancel{2}^{1} \cdot \frac{1}{\cancel{4}_{2}} x^4 + \cancel{6}^{3} \cdot \frac{1}{\cancel{2}_{1}} x^2 + C$

4th :

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

$= \frac{1}{2}x^4 + 3x^2 + C$

Answer: $\frac{1}{2}x^4 + 3x^2 + C$

15) $\int (3x+4)^2 dx$

$(3x+4)(3x+4)$

$9x^2+12x+12x+16$

1st: Rewrite as a FOIL problem and simplify

$$= \int (9x^2+24x+16)dx$$

2nd:

$\int \big(f(x)+g(x)\big)dx = \int f(x)dx \quad + \int g(x)dx$

$$= \int 9x^2 dx + \int 24x dx + \int 16 dx$$

3rd: $\int af(x)dx = a\int f(x)dx$

$$= 9\int x^2 dx + 24\int x dx + \int 16 dx$$

4th :

$$= \overset{3}{\cancel{9}} \cdot \frac{1}{\underset{1}{\cancel{3}}} x^3 + \overset{12}{\cancel{24}} \cdot \frac{1}{\underset{1}{\cancel{2}}} x^2 + 16x + C$$

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int a dx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

$$= 3x^3 + 12x^2 + 16x + C$$

$Answer{:}\ 3x^3 + 12x^2 + 16x + C$

17) $\int (x+1)(x-4)dx$

$x^2 - 4x + 1x - 4$

1st: Rewrite as a FOIL problem and simplify

$$\int (x^2 - 3x - 4)dx$$

2nd:

$\int (f(x) + g(x))dx = \int f(x)dx \quad + \int g(x)dx$

$\int (f(x) - g(x))dx = \int f(x)dx \quad - \int g(x)dx$

$$= \int x^2 dx - \int 3x dx - \int 4 dx$$

3rd: $\int af(x)dx = a\int f(x)dx$

$$= \int x^2 dx - 3\int x dx - \int 4 dx$$

$$= \frac{1}{3}x^3 - 3\cdot\frac{1}{2}x^2 - 4x + C$$

4th :

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ \cancel{x}\ n \neq -1$

Integral of a constant Rule: $\int adx \;=\; ax + C \quad (a\ is\ any\ real\ number)$

$$\frac{3}{1}\cdot\frac{1}{2} = \frac{3}{2}$$

$$= \frac{1}{3}x^3 - \frac{3}{2}x^2 - 4x + C$$

Answer: $\frac{1}{3}x^3 - \frac{3}{2}x^2 - 4x + C$

19) $\int \frac{3x^2+2x}{x} dx$

$$= \int \left(\frac{3x^2}{x} + \frac{2x}{x} \right) dx$$

1[st]: Create two fractions reduce / rewrite

$$= \int (3x+2)\, dx$$

2nd:

$\int (f(x) + g(x))dx = \int f(x)dx \quad + \int g(x)dx$

$$= \int 3x\, dx + \int 2\, dx$$

3[rd]: $\int af(x)dx = a\int f(x)dx$

$$= 3\int x\, dx + \int 2\, dx$$

4[th] :

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ \overset{n}{\cancel{x}} \neq -1$

Integral of a constant Rule: $\int adx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

$$= 3 \cdot \frac{1}{2} x^2 + 2x + C$$

$$= \frac{3}{2} x^2 + 2x + C$$

Answer: $\frac{3}{2}x^2 + 2x + C$

21) $\int 3e^x dx$

$$= 3\int e^x dx$$

1st: $\int af(x)dx = a\int f(x)dx$

$$= 3e^x + C$$

2nd: "e" Rule $\int e^x dx \ = e^x + C$

Answer: $3e^x + C$

23) $\int -\frac{1}{2}e^{x}dx$

$$= -\frac{1}{2}\int e^{x}dx$$

1st: $\int af(x)dx = a\int f(x)dx$

$$= -\frac{1}{2}e^{x} + C$$

2nd: "e" Rule $\int e^{x}dx \ = e^{x} + C$

$answer$: $-\frac{1}{2}e^{x} + C$

25) $\int \frac{7}{x} dx$

$= \int 7x^{-1} dx$

1st: Rewrite with -1 exponent

$= 7 \int x^{-1} dx$

2nd: $\int af(x)dx = a\int f(x)dx$

$= 7 \ln|x| + C$

3rd: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

Answer: $7ln|x| + C$

27) $\int \frac{-4}{x} dx$

$= \int -4x^{-1} dx$

1st: Rewrite with -1 exponent

$= -4 \int x^{-1} dx$

2nd: $\int af(x)dx = a\int f(x)dx$

$= -4 \ln|x| + C$

3rd: "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$

$answer: -4ln|x| + C$

29) $\int 3x^{-1}dx$

$= 3\int x^{-1} dx$

1st : $\int af(x)dx = a\int f(x)dx$

$= 3\ln|x| + C$

2nd : "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$

Answer: $3ln|x| + C$

31) $\int \frac{3}{5} x^{-1} dx$

$= \frac{3}{5} \int x^{-1} dx$

1st : $\int af(x)dx = a\int f(x)dx$

$= \frac{3}{5} \ln|x| + C$

2nd : "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

Answer: $\frac{3}{5} ln|x| + C$